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A proposed method of data transmission from u.h.f. relay stations for remote monitoring

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A PROPOSED METHOD OF DATA TRANSMISSION FROM U.H.F. RELAY STATIONS FOR REMOTE MONITORING

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Summary

This report examines a new system to provide a data link between a u.h.f. television relay station and a monitoring and information centre. It shows that data can be transmitted with the broadcast signal and received outside the normal service area. There is a brief description of a practical test of the link.

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1. Symbols

a	The r.f. level of the logical one signal as transmitted as a fraction of the level of the sync. pulse.
A_e	Effective area of an aerial.
b	The r.f. level of the logical nought signal as transmitted as a fraction of the level of the sync. pulse.
C_e	Chances of finding the framing code erroneously.
F	Noise figure of the receiver.
G_o	Gain of an aerial with respect to an isotropic radiator.
h	Eye height of the received data.
k	Boltzmann's constant.
m	Number of times the receiver looks for the framing code before suppressing the output.
n	A variable.
P_e	Probability that any one bit is in error after error correction.
P_o	Probability that the receiver will have incorrectly located the framing code whilst giving an output.
R_r	Radiation resistance of the aerial.
$\mathbf{S}$	The Poynting vector.
$S_o(m)$	Chances of not finding the framing code after m searches when the receiver is out of lock.
$S_i(m)$	Chances of finding the framing code after m searches when the receiver is in lock.
T	Noise temperature of the receiver.
$\overline{t_i}$	Mean time the receiver will continue operating in lock.
$\overline{t_o}$	Mean time the receiver will continue operating out of lock.
u	Square root of the signal-to-noise ratio at the input to the receiver.
V	The field-strength at the receiver (V/m).
V_n	A value of the noise voltage.
V_p	The amplitude of the data from mean to peak.
V_{pk}	The signal level at the peak of the syncs.
W_n	Available noise power.
W_s	Available signal power.
x	Probability of any one bit being in error before correction.
y	A dummy vehicle.
Δf	The noise bandwidth of the receiver.
$\epsilon(n)$	The probability of n errors in an eight bit word before correction.
$\Phi(u)$	The normal distribution integral.
λ	Wavelength of radiation.
σ_n	r.m.s. noise voltage.

2. Introduction

The BBC is improving its facilities for monitoring its transmitters and relay stations. Operation at most broadcasting sites is now fully automatic. This means that there must be an efficient monitoring service to help the maintenance teams to locate and diagnose any fault.

The facilities of the monitoring service are based on monitoring and information centres[1] (MICs). These are places where information from each broadcasting site is collected. This information is used to arrange the work of each maintenance team; it could be either routine maintenance or an emergency visit.

Each important piece of equipment on a broadcasting site is monitored for faults. Any piece of equipment which is crucial to the service will be monitored (e.g. main supplies, programme feeds, amplifying equipment, burglar alarms etc.). There is a connection from each piece of equipment to the fault reporter. This usually carries a signal saying the device is working correctly or incorrectly.

Whenever there is a significant change in operating conditions the fault reporter sends a message to the MIC. The significant change can be either a fault developing or a fault clearing. Either is of interest to the maintenance team. If the information is sufficiently important the maintenance team may be called out straight away. Alternatively the fault may be minor and can be left until the next routine visit.

The message from many sites is usually sent to the MIC by telephone. In other cases, with modulating transmitters, the message can be modulated on to a pilot tone. This is then added to the sound channel at baseband and broadcast as usual. When the signal reaches the MIC it is demodulated and decoded. For some sites there is no feed of baseband signal or the MIC is not in their service area. For these sites the information is sent by telephone instead whenever practicable.

The telephone link is not always a satisfactory link. The reasons for this are mainly economic. Firstly, there is a high installation charge at many of the sites which are now being commissioned. Secondly, the cost of a call limits the time for which the link may be in use. In practice a change of state must persist for a time before a call is made. The exact length of time, which may be between 30 seconds and 26 minutes, depends on local conditions. This delay has the advantage of preventing calls which are due to momentary faults. It will also reduce the number of calls due to any apparatus working very close to its threshold. On the other hand, it is undesirable for a complete shut-down in service to be undetected for 26 minutes. The delay must be a compromise between cost and speed.

Instead of using a telephone line we can transmit the information with the television signal. The information can be transmitted as binary pulses in the field-blanking interval. For relay stations this appears to be more practical than

using pilot-tone methods. The proposed system uses pulses similar to those used in CEEFAX but of a much lower bandwidth. As there is no feed of baseband at the relay station, the data must be added to the signal at radio frequency. It is possible to do this using a reflection modulator.[2]

As the MIC will not always be in the service area of the transmitter, the data must be rugged. The signal at the MIC may be weak and full of errors. The data must be sent in such a way that it can be corrected.

The remainder of this report describes a way of transmitting data with the television signal over moderate distances. It states the requirements of the MIC and then describes the format of the data, with the method of transmission and reception. This is then followed by a discussion of the causes and correction of errors in the system. This leads to an expression for the minimum field-strength required at the MIC for good reception.

3. The specification of the link

The link between the u.h.f. relay station and the MIC must be capable of passing information accurately. At the relay station the problem is how to interface between fault reporter, modulator and transposer. At the MIC the major problem is error correction.

At the u.h.f. relay station the fault reporter supplies data at the rate of one 8-bit word every 400 ms. Each word is available in serial or parallel form. The modulator must take this information and add it to the u.h.f. signal without any significant reduction in quality of the u.h.f. signal for television reception.

At the receiving end the data must be presented to the MIC accurately. The present specification is that the data must have less than one error in 10^4 bits.

4. The transmission channel and data

In this proposal, the information is transmitted with the 625-line broadcast television signal in the field-blanking interval.

4.1. The channel

The information is transmitted as binary pulses on lines 15 and 328. The pulses are added to the u.h.f. signal at the input of the relay station by a reflection modulator.[2]

Lines 15 and 328 are the lines available. Other data systems (CEEFAX, ITS, etc.) use lines 16 and 329 upwards. Earlier unpublished work gives the impression that data on lines 1 to 14 and 314 to 327 will be visible on fly-back in old sets. This leaves lines 15 and 328 as the only ones now available for transmitting data.

The pulses of data are added to the u.h.f. signal at the input to the transposer. The equipment to do this is an add-on box. This fits between the aerial which receives the incoming signal and the input to the transposer itself.

The pulses of data are added by a reflection modulator. This type of modulator has been fully described in an earlier report.[2] With this modulator the maximum depth of modulation is about 50%. It enables us to add data to a signal at u.h.f.

The specification of the u.h.f. television channel allows a maximum bandwidth of the data of 1·25 MHz. This is because the modulator adds a double-sideband signal and the relay station must radiate a signal with a vestigial sideband which starts to cut off at a frequency 1·25 MHz below the frequency of the vision carrier. For frequencies of modulation less than 1·25 MHz we can transmit a double sideband. Anything requiring more bandwidth cannot be transmitted in this way. (It is not practicable to incorporate a full vestigial sideband filter, bearing in mind that the incoming television signal has already been correctly filtered at the parent station.)

This means the pulses of data are limited in the time domain and the frequency domain. Data can only be sent on two lines every 625, i.e. for 0·25% of the time, and its bandwidth is restricted to 1·25 MHz.

4.2. The format of the data

There are many possible forms of coding from which to choose for the system.

In the 400 milliseconds between each word coming from the fault reporter, it is possible to transmit between 20 and 2500 bits of information. In the 400 milliseconds we can put data on lines 15 and 328 twenty times. The minimum number of bits on one line is one. This leads to a minimum rate of transmission of 20 bits for every 8-bit word from the fault recorder. On the other hand we can add data at $2{\cdot}5 \times 10^6$ bits per second during the line period at the maximum frequency for modulation of 1·25 MHz. With this rate twenty lines with an active period of 50 μs give 2500 bits for each word.

So there is a wide range of possible transmission rates which can be used. It is possible to transmit the data slowly. This gives a system with low bandwidth and which is relatively noise free. Because there is little noise there will be few errors. There will, on the other hand, be less possibility of any correction of errors. If the data are transmitted more quickly, the bandwidth will be wider and there will be more noise. This will in turn lead to more errors. These errors could be corrected by using redundancies in the data.

A coding which is easy to engineer is to put each word from the fault reporter once on every line 15 or 328. This would give a total of twenty repetitions of each word. Also, having eight bits on each line gives a low data rate and bandwidth. This means that the system would be relatively immune to noise. It also has a great deal of redundancy and so errors can be corrected.

The different words must be separated by a framing code. If they are not, any error corrector will be unable to tell when a new word starts after a period of repetition of the previous word. The framing code could be a code common to all fault reporters. Alternatively, it could be used as an identification for individual relay stations. The framing code is an 8-bit word like the data. This allows the receiver to process the data and framing code in the same way.

The suggested format is thus the output from the fault reporter repeated 19 times followed by a framing code. This is both easy to engineer and effective.

5. The receiver

At the MIC there must be a receiver which will recover the data as free as possible from errors. This must be achieved in spite of all sources of errors that may be introduced.

In summary, the characteristics of the incoming data are as follows.

Type of data	=	binary pulses
Position of data	=	lines 15 and 328
Data rate	=	8 bits per active line
Clock rate	=	twelve times line frequency
Bandwidth on transmission	=	±1·25 MHz.

The receiver must demodulate the incoming u.h.f. signal, convert the data from its analogue form to digital, and then process the data to remove errors.

5.1. Causes of errors

The MIC is not usually in the television service area of the relay stations that it monitors. The signal received at the MIC is, therefore, often very weak and subject to various forms of degradation, which may cause errors in the received signal.

The four main causes of errors are:

1. Noise
2. Ghosting, i.e. multipath propagation
3. Co-channel interference (c.c.i.)
4. Timing jitter in the receiver.

There will always be noise present on the signal. Man-made noise at u.h.f. is normally low and can be ignored. Therefore, the noise that degrades the received data will usually be thermal in character. We need to consider the noise temperature of the receiving aerial and noise generated in the early stages of the receiver.

Ghosting may occur on any transmission path. It may be constant, caused by hills, high buildings etc. Alternatively, it may be subject to fading. This occurs most noticeably in the form of 'aircraft flutter'. In some instances it is possible that the delayed version of the signal may be stronger than the direct signal.

Co-channel interference can be prominent in many cases. Relay stations are designed so that they radiate to a particular area: the service area. Inside the service area there should be insignificant amounts of c.c.i. Outside the service area the signal will not necessarily be protected from c.c.i. This may be the case at the MIC. The interference from c.c.i. is likely to fluctuate, often depending on weather conditions.

Timing jitter will occur if the timing in the receiver is derived from a noisy signal. The main problem is to know when to look for the data. This knowledge is derived from the synchronizing pulses of the received signal. If the received signal is poor there will be jitter in the timing circuits. This could lead to the wrong data being recovered.

Summarizing the situation there will, in general, be interference giving rise to a distorted wave-form. In particular there may be ripples caused by c.c.i., and also overshoots, undershoots and delayed pulses arising from ghosting. On top of this will be noise. All these factors will control the possible occurrence of errors.

The distortion can be thought of in terms of a reduction in the eye height of the signal. This is the separation between 0 and 1 demodulated signal levels in the worst case. As interference is added to the signal, the eye height is reduced. When the eye height is reduced to zero the signal is no longer recoverable. The reduction in eye height may be taken in two parts. Firstly the degradation due to c.c.i., ghosting and any timing jitter will give a fairly constant reduction in eye height. The noise will then worsen this reduction, sometimes closing the eye and causing errors. A simple analysis of the errors expected can be made by assuming the signal is a normal digital signal with magnitude equal to the reduced eye height. Then when random noise is introduced, errors are added as in standard theory.[4]

Clearly, the number of errors depends on both the eye height of the signal and the amount of noise present. The error rate is calculated in Appendix IV.

5.2. Ways of reducing errors

It is possible to reduce errors to a minimum by careful engineering, particularly in the aerial system and receiver.

Noise can be reduced by using an aerial with high gain and in some cases an amplifier with a low noise factor. A high-gain aerial should help to keep noise to a minimum by raising the signal level. A pre-amplifier may give a similar result. In practice, amplifiers may overload if there is a signal at high level in the area. Most MICs will be near a main transmitter which will be radiating at high power levels. Some of this signal could be picked up by the receiving aerial. If the level is too great the pre-amplifier will overload. So, pre-amplifiers must be used carefully.

The amount of noise is dependent on the bandwidth of the receiver. The energy of thermal noise is directly proportional to the bandwidth. A low bandwidth thus

implies low noise. The lower limit is set by the effect of filtering on the data. If the bandwidth is too low then there will be interference between different bits of the data. Nyquist has derived a limit on the bandwidth for this not to occur: the bandwidth (at baseband) must be at least equal to half the clock rate of the data. In our case this limit is 93·75 kHz. In practice there may be group delay associated with the filter. This too can cause interference between bits of data. Its effect can be reduced by correction or by widening the bandwidth. This latter alternative is only feasible if noise is not too much of a problem. The effect of filtering is most predictable if the system's bandwidth is defined by only one filter. In our system the bandwidth on transmission is relatively wide (±1·25 MHz). It can be limited accurately within the receiver. This is best done before detection if an envelope detector is used: after detection is more convenient if a synchronous detector is used.

Co-channel interference can be reduced by careful siting of the aerial. Often c.c.i. will arrive mainly from one fixed direction. If this is the case it is a simple matter to arrange a minimum in the radiation pattern of the aerial in that direction to reduce c.c.i. In cases where c.c.i. arrives from various directions, a multiple-aerial array or an adaptive array[5] may be used. These may be expensive when compared with transmission by telephone line.

Ghosting may also be reduced by a careful siting of aerials. This can reduce some ghosts, but it is not easy to remove all of them. Once ghosts are on the signal they are difficult to remove by electronic processing.

The effects of timing jitter can be removed by using another transmission as the source of synchronizing pulses. Timing pulses can be derived from the incoming signal. However, each relay station is fed with signal by a re-broadcast link. This involves receiving the signal from a nearby transmitter, changing the frequency and then re-radiating it. The signal radiated by the relay station should be identical to the signal radiated by the main transmitter but with a time lag. In our case, the MIC will often be near that main transmitter. If so, we can use the programme radiated from the main transmitter as a source of timing information. This should be undistorted at the MIC. All we need to do is allow for the delay between the two signal paths: one direct to the MIC from the transmitter; the other via the relay station.

Thus we can reduce errors by careful engineering of the receiver. We cannot eliminate them however.

5.3. Error correction

There are two effects of errors: either the data will be wrong or the framing code will be wrong. The data can be corrected once the framing code has been found. Until the framing code is found the output from the receiver must be suppressed to avoid errors.

5.3.1. Mechanism for error correction

The flow diagram for error correction is shown in Figure 1. It takes no account of the starting procedure. Assuming that the framing code has been correctly located, there is an output of data beginning when the start code (all zeros) has been received. The output of the corrected data continues whilst the framing codes are still found in the right place.

If a sequence of framing codes is not detected the receiver inhibits the output. This is because when the framing code is not being received correctly the data is most likely to be wrong too.

Then the receiver scans through the incoming words of data until the framing code is detected. The flowchart shows that every nineteenth word is scanned for the framing code. There is no reason why every word should not be scanned for the framing code. At the time of the experimental work it was more convenient to engineer a system scanning every nineteenth word.

Once the start code has been received after a framing code the machine starts its output again.

5.3.2. Finding the framing code

The problem is to find the framing code in the presence of errors.

If the receiver looks for the framing code, five things can happen:

1. The framing code will be found
   (a) because it was transmitted and received correctly
   (b) because data was transmitted and received wrongly

2. A data word will be found
   (a) because it was transmitted and received correctly
   (b) because a different data word has been received incorrectly
   (c) because the framing code has been received incorrectly.

Each of these occurrences will have a certain probability. We are only interested in 1(a) and 1(b) at the moment.

We must distinguish between 1(a) and 1(b). If the framing code has been correctly found we can continue to allow an output of data. If the framing code has been found in error we must consider that the data is also wrong and suppress the output.

If we look every 20 words, we should keep seeing the framing code if 1(a) occurred in the first instance, but not when 1(b) occurred. This procedure works well with a small number of errors as the difference between 1(a) and 1(b) is then well marked. The results in Appendix II show that with few errors occurring we nearly always detect the framing code again if we have once detected 1(a) and hardly ever if we have detected 1(b). The code in the next framing code's position should tell us whether our first detection was an instance of 1(a) or 1(b).

As the number of errors increases, the method becomes less reliable. With more errors the framing code

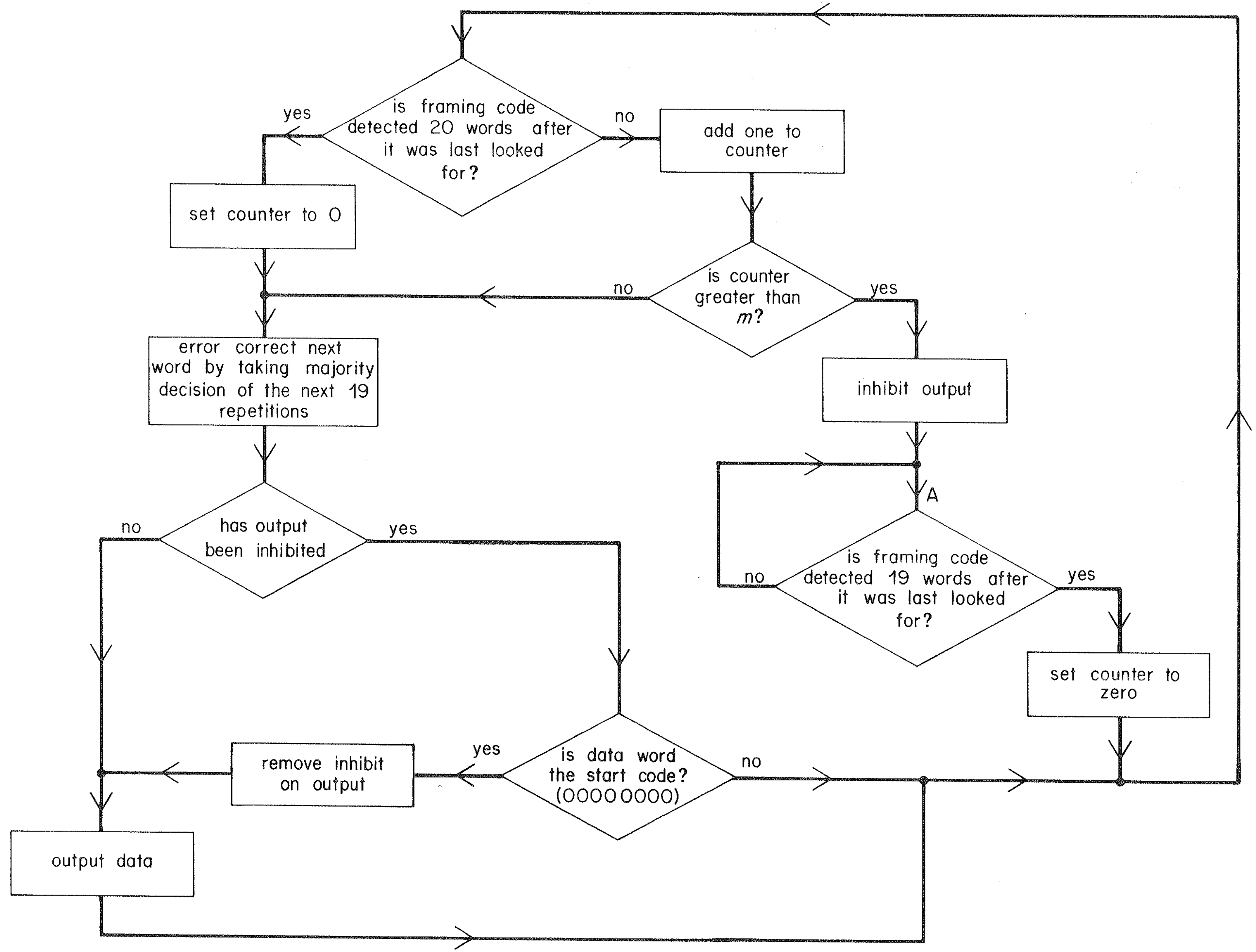

Fig. 1 - Flowchart of the data processing

To understand the process it is best to enter the loop at Point A. Assume that a framing code was not found last time it was looked for and that the output has been inhibited

occurs less often, even if we had a valid framing code in the first place. This is shown on Appendix II. This means we must look more times for the framing code. As we do this, the chances of an incorrect framing code increase and with it the chance that the system will start up incorrectly. Once more, Appendix II derives the formulae and gives examples.

The length of time we look for the framing code is dictated by the error rate we want for the output. It is possible to calculate the mean time the receiver will be giving correct and incorrect outputs. This is done in Appendix III. By setting a limit on the ratio of these quantities we fix the number of times we can look for the framing code in any time slot. This depends on the initial error rate as well as the desired error rate at the output. Appendix III quotes two examples. We can see we do not have to look long even for high error rates. For signals with an initial error rate of 1 in 10 we need not look more often than about twelve times to obtain an output error rate of 1 in 10^4. For better signals this time is reduced.

Once the framing code has been found the next step is to look for the data.

5.3.3. Correcting the data

The data are sent using a highly redundant code so it is possible to correct a lot of errors. It is reasonable to assume that when a signal is repeated 19 times there must be some way of correcting it when errors are present.

The easiest and best method is to use a majority decision. Any single bit in a word should be received 19 times. If it is in error occasionally the bit will be received less often. The number of times the bit is received as a one is then compared with the number of times it is received as a nought. If it is a one most often the receiver assumes that is its correct value and vice-versa.

Majority decision will improve the error rate by several orders of magnitude. Figures for this are given in Appendix I. For instance, an original error rate of 1 in 10 can be improved to about 1 in 10^5. An original error rate of 1 in 100 will be improved to about 1 in 10^{15}. As this is about one error every two million years it is somewhat academic. Other causes of errors will be more important.

6. Field-strength wanted at the receiver

Having decided upon an error rate which is acceptable what is the minimum field-strength required at the receiver? The errors caused by ghosting, c.c.i. and timing jitter can be thought of in terms of a reduction in eye height. In general the reduction in eye height will be unpredictable, but measurable. The receiver will then add noise to the signal. This noise will be predictable and measurable.

Appendix IV derives the relation between error rate, eye height and field-strength.

As the amount of interference increases, the amount of noise which is permissible for good reception decreases. If, for instance, there is no interference other than random noise the system may work at places with a field-strength as low as 26 dB(μV/m). For every decibel that the eye height is reduced by interference, either the field-strength must be increased by the same amount or the noise figure decreased by the same amount. An acceptable field-strength in practice will probably be 35 dB(μV/m), as a very rough estimate.

7. Experimental trial of the link

An experiment was performed to determine whether this type of link was feasible. The tests had to show that data could be transmitted accurately over a typical path. They also had to show that the data did not interfere with the broadcast signal.

The path chosen was between Reigate Hill on the North Downs and BBC Designs Department in Central London. Reigate Hill is an early type of relay station. It is intended to serve the people living just to the south of the North Downs. Because of this there is little radiation in the direction of London. The path from Reigate Hill to

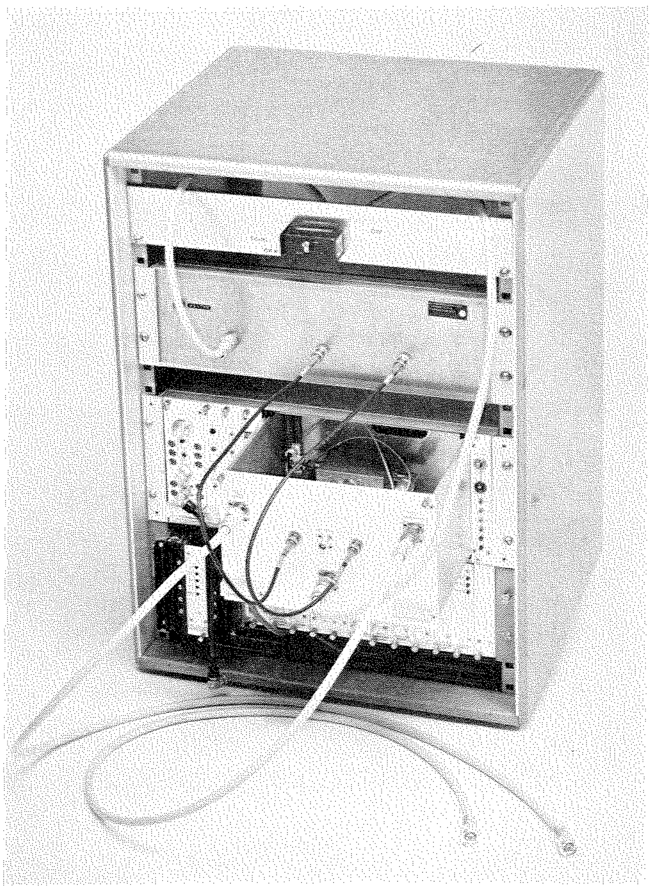

Fig. 2 - The modulator and fault reporter installed in an equipment rack. The two unconnected leads are the input and output

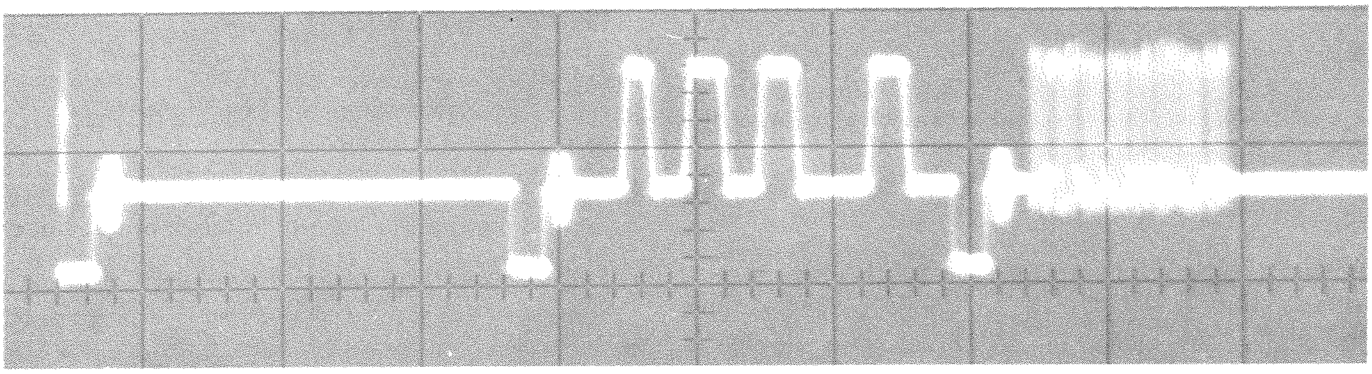

Fig. 3 - The output from the modulator, showing line 14, line 15 with data on and line 16 with ICE

London would be line of sight but for the large buildings in Central London. One building in particular blocked the direct path.

The modulator was installed in the u.h.f. transposer at Reigate Hill for a period of two months. During this time it radiated a fixed sequence of codes on lines 15 and 328 of the BBC-2 transmission. Figure 2 is a photograph of the modulator and fault reporter together in a small frame. Figure 3 is a photograph of some data on line 15.

During the two months of the experiment there were no complaints from the public. This indicates that few, if any, sets suffered from troubles associated with slow fly back during the field-blanking interval. Lines 15 and 328 are satisfactory for the transmission of data therefore.

For part of the two months a receiver was installed at the site in Central London and worked satisfactorily. This received a noisy but usable signal from Reigate. The receiver used a conventional u.h.f. tuner and synchronous detector. The i.f. filtering was closer to double sideband than vestigial sideband. The signal was filtered to remove the noise by a 200 kHz low pass filter after detection. This removed most of the noise and added little of the distortion caused by group delay. The data was then recovered from the video signal, error corrected and passed to the MIC.

The output from the receiver was mostly error free. This was checked by displaying the output on a set of eight lights. The sequence of light patterns could, with practice, be compared with the expected output. When errors did occur they usually occurred in bursts. In most cases the output was suppressed shortly afterwards.

There were no major problems encountered during the experiment. There are u.h.f. receivers in both the transmitting and receiving terminals of this system. In the experiment they tended to drift off frequency over a period. This is a minor problem and can be overcome by better design. The second problem was the interface between the receiver and the MIC. This caused a few errors but again should be overcome by a correctly designed interface. Because of this we could not analyse the errors in the received signal. There is no accurate figure for the error rate therefore.

During the experiment the signal level at the input to the receiver was 7 μV. This corresponds to a field-strength at the aerial of about 45 dB(μV/m) when the cable loss and aerial gain are added. The signal level was lower than predicted by conventional computation, but is fully explained by the presence of a large building which blocked the direct path. This reduced the main signal to one propagated by refraction round the building so that the principal component of the signal arrived from a direction coinciding with the Crystal Palace transmitter 31° away from the actual bearing. The level of signal from Crystal Palace was too high for us to put a pre-amplifier in the aerial circuit.

The signal did sometimes suffer from c.c.i. but not from ghosting. The c.c.i. was most probably generated by the Oxford transmitter. It was clearly visible on an oscilloscope trace of the signal, but did not create a noticeable increase in errors. If any ghosting were present it was not visible above the noise on the signal.

The results of the experiment were encouraging. Data were successfully sent from Reigate Hill to London. There was no noticeable degradation of the normal television service. This shows that the link is a practical proposition.

8. Conclusions

It is possible to transmit data from a transposer to a monitoring and information centre using an 'on air' channel with the following specification:

Transmission channel	:	lines 15 and 328 of the transmitted t.v. signal
Rate at which data is supplied	:	One word of eight bits every 400 ms
Transmission rate	:	187·5 kBaud
Error rate at the output	:	better than 1 in 10^4
Theoretical minimum field-strength in ideal surroundings	:	About 26 dB(μV/m)
Practical field-strength for reasonable operation	:	About 35 dB(μV/m)

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Appendix I

Improvement of the error rate by majority decision

If a given bit is transmitted 19 times it must be received wrongly at least 10 times before the majority decision circuits give that bit the wrong value. The probability of a bit having n errors in 19 repetitions is the (n+1)th term in the binominal expansion of $((1-x)+x)^{19}$. The probability of the bit being in error after correction is given by the sum of the last ten terms in the expansion, as follows.

$$P_e = \sum_{n=10}^{19} \binom{19}{n} (1-x)^{(19-n)} x^n$$

TABLE 1

Error rate after correction (P_e) compared with error rate before correction (x)

x	P_e
0·01	$8{\cdot}5 \times 10^{-16}$
0·05	$5{\cdot}9 \times 10^{-9}$
0·10	$4{\cdot}0 \times 10^{-6}$
0·15	$1{\cdot}4 \times 10^{-4}$

Appendix II

Chances of finding the framing code

If the receiver thinks it is in lock it will look for the framing code every 20th word. If it continues to find it, the receiver will continue giving an output. If it fails for more than a certain number of times it will assume it is out of lock and suppress the output.

If the system is in lock the framing code may not be detected because of errors. We wish to know the chances of this happening.

If the system is not in lock the framing code may be detected because of errors. We want to know the chance of this happening too.

The chances of not finding the framing code when the system is in lock

When the system really is in lock we must find the probability of no errors in at least one of m samples.

The probability of no errors in an eight bit word is given by:

$$\epsilon(0) = (1-x)^8$$

Therefore the probability that the transmitted code will not be seen at least once by the mth repetition is given by:

$$S_i(m) = [1-(1-x)^8]^m$$

This is evaluated in Table II.1.

Chances of finding the framing code when the system is not in lock

Now take the case where the framing code has been erroneously detected. We must see whether it will be found again in m samples.

The chances of errors occurring in a word are $1-\epsilon(0)$.

TABLE II.1

Chance of not finding the framing code at least once in m repetitions ($S_1(m)$)

m \ x	0·1	0·05	0·01
1	$5{\cdot}7 \times 10^{-1}$	$3{\cdot}4 \times 10^{-1}$	$7{\cdot}7 \times 10^{-2}$
2	$3{\cdot}2 \times 10^{-1}$	$1{\cdot}1 \times 10^{-1}$	$6{\cdot}0 \times 10^{-3}$
4	$1{\cdot}1 \times 10^{-1}$	$1{\cdot}3 \times 10^{-2}$	$3{\cdot}6 \times 10^{-5}$
8	$1{\cdot}1 \times 10^{-2}$	$1{\cdot}6 \times 10^{-4}$	$1{\cdot}3 \times 10^{-9}$
16	$1{\cdot}2 \times 10^{-4}$	$2{\cdot}7 \times 10^{-8}$	$1{\cdot}6 \times 10^{-18}$

If the errors are random the chance of any one word being generated is the same for each word.

The chance of finding the framing code erroneously is thus given by:

$$C_e = \frac{1 - \epsilon(0)}{256}$$

This leads to the probability that the framing code will not be detected erroneously after m repetitions.

$$S_o(m) = \left(1 - \frac{1 - \epsilon(0)}{256}\right)^m$$

The chances of finding it erroneously are $1 - S_o(m)$.

TABLE II.2

Chance of finding a false framing code in m repetitions ($1 - S_o(m)$)

m \ x	0·1	0·05
1	0·002	0·001
2	0·004	0·002
4	0·009	0·005
8	0·018	0·010
16	0·036	0·021

Appendix III

Probability that the receiver will give an output whilst out of lock

This probability is defined as:

$$P_o = \frac{\text{Time giving an output whilst not locked to the framing code}}{\text{Time giving an output}}$$

The numerator and denominator can be calculated separately. They are each dependent on two factors. These factors are, firstly, the chance of starting either in or out of lock, and secondly, the mean length of the sequence before the output is stopped.

The time the receiver is in lock is calculated first. If the receiver looks for the framing code at random it will look in the correct place with a probability of 1/20. The change of there being no errors in the framing code is $\epsilon(0)$. The chance, therefore, that the receiver finds the framing code correctly is $\epsilon(0)/20$. Once the receiver is giving an output it will continue for a mean time of $0{\cdot}4\ m/S_i(m)$ seconds. Thus the mean time the system will continue in lock from any one search is:

$$\bar{t}_i = \frac{\epsilon(0)}{20} \times \frac{0{\cdot}4m}{S_i(m)}$$

Then the time the receiver is out of lock is calculated. If the receiver looks for the framing code at random it will look in the wrong place with a probability of 19/20. The chance that this bit of data is wrong in such a way that it gives the framing code is shown in Appendix II to $(1-\epsilon(0))/256$. The chance that the receiver will start up is thus:

$$\frac{19}{20} \cdot \frac{1-\epsilon(0)}{256}$$

Once the receiver is giving an output it will continue for a mean time of $0{\cdot}4\ m/S_o(m)$ seconds. Thus the mean time the system will continue out of lock from any one search is:

$$\bar{t}_u = \frac{19}{20} \cdot \frac{1-\epsilon(0)}{256} \cdot \frac{0{\cdot}4m}{S_o(m)}$$

These two results lead to the probability that the receiver will be working incorrectly if

$$P_o = \frac{\bar{t}_u}{\bar{t}_i + \bar{t}_u}$$

$$= \frac{19 \,.\, S_i(m) \,.\, (1-\epsilon(0))}{19 \,.\, S_i(m) \,.\, (1-\epsilon(0)) + 256 \,.\, S_o(m) \,.\, \epsilon(0)}$$

We expect to choose the number of samples m so that the receiver will soon realise it is out of lock. In other words, $S_o(m)$ has a value very close to one. Also $S_i(m)$ is negligible. This simplifies the expression for P_o:

$$P_o = \frac{19}{256} \cdot \frac{1-\epsilon(0)}{\epsilon(0)} \,.\, S_i(m)$$

This expression leads to a minimum value for m for a given error probability. If the receiver starts up incorrectly it must stop as soon as possible. This means that m must be as small as possible. This expression for P_o gives that minimum value.

For example, we have a defined error rate of 1 in 10^4, i.e. $P_o < 10^{-4}$. If we choose an original error rate for the data of $x = 0{\cdot}1$, then:

$$S_i(m) < 10^{-3} \quad \text{or} \quad m > 12{\cdot}25 \quad \text{from Appendix II}$$

If we choose an original error rate of 0·05, then $m > 5{\cdot}5$.

Appendix IV

Field-strength required for a given error rate

In this Appendix the signal and noise levels in the receiver are manipulated to give an equivalent signal-to-noise ratio for the binary signal. This ratio is then used to determine the error rate.

The first step is to work out the signal level. The power available at the terminals of an aerial is given by:

$$W_s = A_e \cdot S$$

$$= \frac{G_o \lambda^2}{4\pi} \cdot \frac{V^2}{120\pi}$$

For an aerial with a radiation resistance R_r and under peak detection, the voltage recovered will be

$$V_{pk} = \frac{\lambda V}{2\sqrt{2}\,\pi} \sqrt{\frac{4G_o R_r}{30}}$$

The digital signal will fluctuate between two levels, $a\,V_{pk}$ and $b\,V_{pk}$. The signal will have a mean value of $\frac{1}{2}(a+b)\,V_{pk}$ and a magnitude of $\frac{1}{2}(a-b)\,V_{pk}$, from its mean level to its peak. This signal is given by:

$$V_p = \frac{(a-b)\lambda V}{4\sqrt{2}\,\pi} \sqrt{\frac{4G_o R_r}{30}}$$

The next step is to work out the amount of noise. The noise generated in the receiver is given by:

$$W_n = FkT\Delta f$$

where W_n is referred to the input. This is equivalent to a source of noise voltage of r.m.s. value.

$$\sigma_n = \sqrt{4FkT\Delta f R_r}$$

in series with the impedance of the aerial.

Hence the signal-to-noise ratio of the digital signal is

$$\left(\frac{V_p}{\sigma_n}\right)^2 = \left(\frac{(a-b)\lambda V}{4\sqrt{2}\,\pi} \cdot \sqrt{\frac{4G_o R_r}{30}}\right)^2 \cdot \frac{1}{4FkT\Delta f R_r}$$

$$= \frac{G_o \lambda^2}{4\pi} \cdot \frac{V^2}{120\pi} \cdot \frac{(a-b)^2}{2} \cdot \frac{1}{FkT\Delta f}$$

The probability of error in a digital signal depends on the amount of noise and is given by:

$$x = \frac{1}{\sqrt{2\pi}\,\sigma_n} \int_{V_p}^{\infty} \exp(-V_n^2/2\sigma_n^2)\, dV_n$$

$$= \frac{1}{\sqrt{2\pi}} \int_{V_p/\sigma_n}^{\infty} \exp(-y^2/2)\, dy$$

$$= 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{V_p/\sigma_n} \exp(-y^2/2)\,dy$$

But $$\left(\frac{V_p}{\sigma_n}\right)^2 = \text{signal-to-noise ratio}$$

$$\therefore \quad x = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp(-y^2/2)\,dy$$

$$= 1 - \phi(u)$$

where $$u = V_p/\sigma_n$$

$$= \left(\frac{G_o\lambda^2}{4\pi} \cdot \frac{V^2}{120\pi} \cdot \frac{(a-b)^2}{2} \cdot \frac{1}{FkT\Delta f}\right)^2$$

and where $$\phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp(-y^2/2)\,dy$$

$\phi(u)$ is the normal distribution integral which is fairly well tabulated.[6]

If the eye height is included in the calculation, the limits V_p/σ_n of the integral are modified to hV_p/σ_n. Following through gives:

$$u = h\left(\frac{G_o\lambda^2}{4\pi} \cdot \frac{V^2}{120\pi} \cdot \frac{(a-b)^2}{2} \cdot \frac{1}{FkT\Delta f}\right)^{1/2}$$

For example, with

G_o = 10 dB
λ = 0·5 m
a = 0·76
b = 0·38
Δf = 200 kHz
F = 10
T = 300 K
h = 1·00

then, $u = 6{\cdot}78 \times 10^4 \times V$

For given values of V we can calculate u and hence the error rate of x.

x	u	V	V = dB(μV/m)
0·1	1·28	$18{\cdot}9 \times 10^{-6}$	25·5
0·05	1·64	$24{\cdot}3 \times 10^{-6}$	27·7
0·01	2·32	$34{\cdot}2 \times 10^{-6}$	30·7
0·0001	3·71	$54{\cdot}8 \times 10^{-6}$	34·8
10^{-6}	4·75	$70{\cdot}0 \times 10^{-6}$	36·9

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